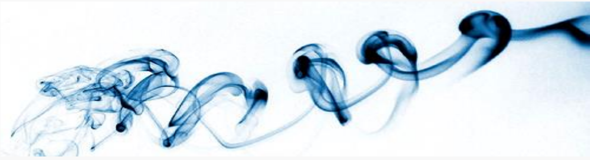

ME 321: Fluid Mechanics-I

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Lecture - 03 (26/04/2025)

Fluid Dynamics: Basic concepts & Kinematics

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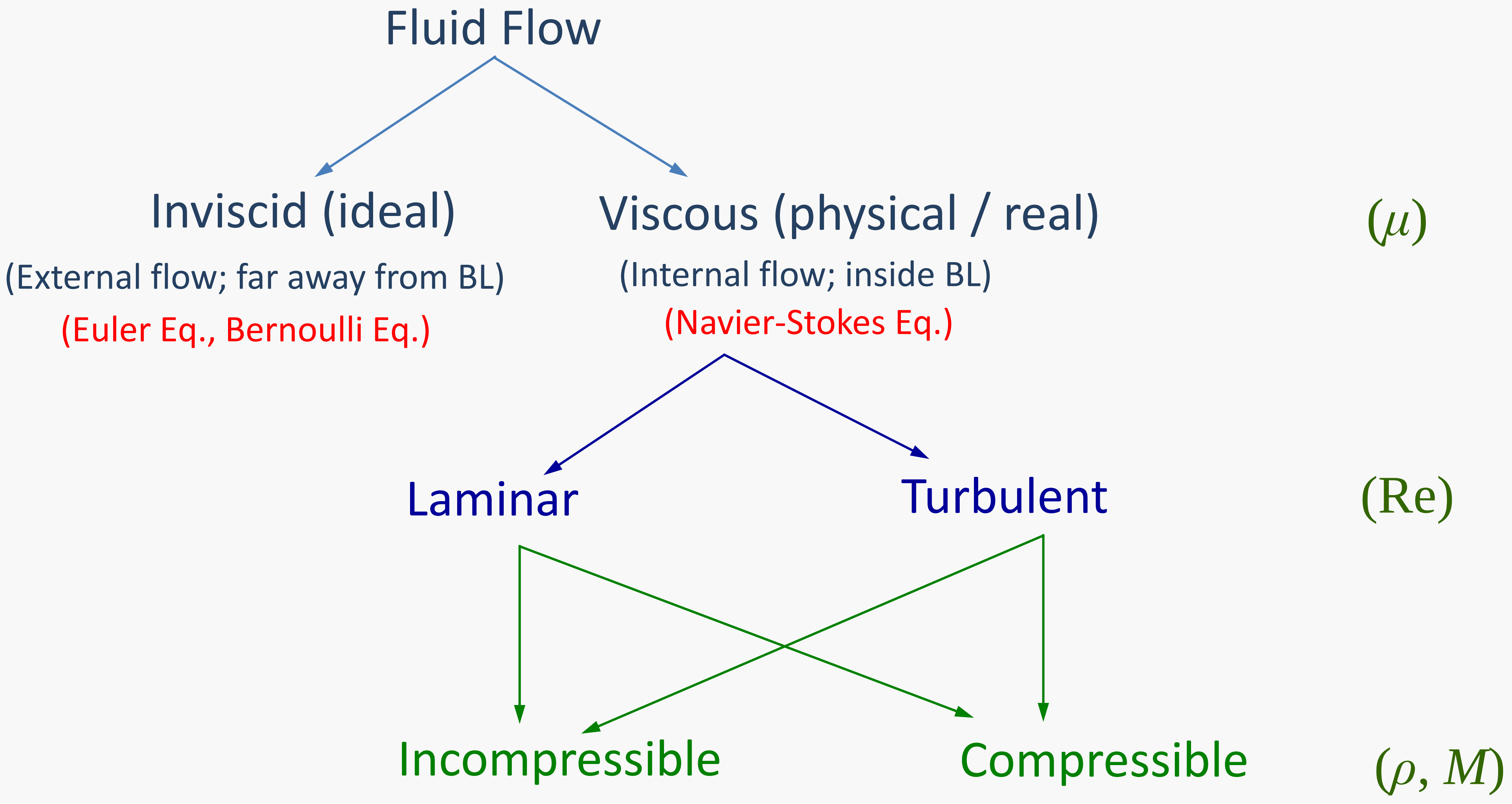
Analysis of flow of fluids can be performed in many ways. A simple example of continuum fluid flow classifications for the purpose of their analysis is shown below:



Fluid Flow

Inviscid (ideal)

(External flow; far away from BL)

(Euler Eq., Bernoulli Eq.)

Viscous (physical / real)

(Internal flow; inside BL)

(Navier-Stokes Eq.)

(μ)

Laminar

Turbulent

(Re)

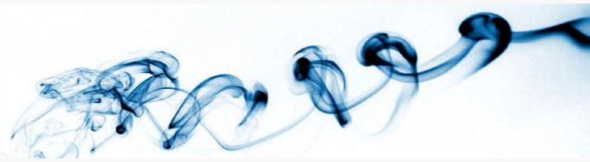
Incompressible

Compressible

(ρ, M)

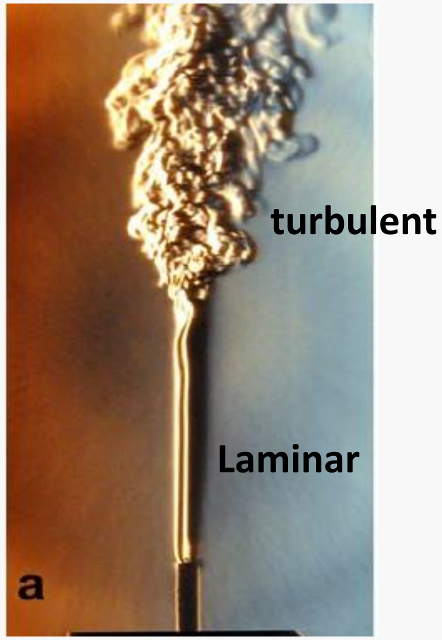
(BL = Boundary layer (ME 323))

Laminar and Turbulent Flows



Laminar flow occurs when a fluid flows in parallel layers and slide past one another. This flow is very **regular, well-behaved and smooth**. There will be no lateral mixing and interaction between the layers. In classical scale, laminar flow occurs at low speeds with low Re.

Turbulent flow is a fluid motion with particle trajectories varying randomly in time, in which **irregular fluctuations** of various flow properties arise.

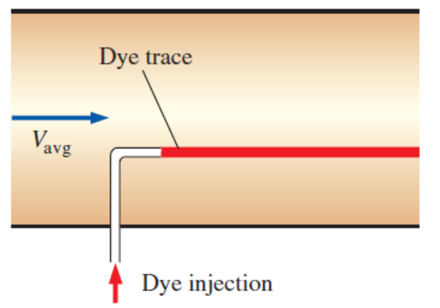


Candle plume

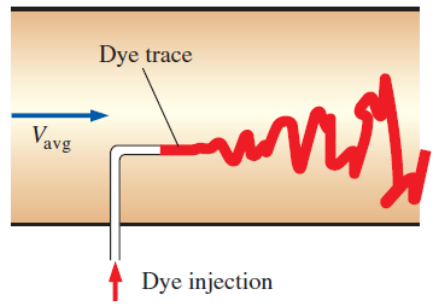
Due to conditions imposed by the geometry and flow field, such as

- surface topography
- surface roughness
- pressure gradient
- surface mass injection (or suction)
- surface temperature and so on,

the interaction of the fluid particles increases and takes place at the macroscopic level; the **streamlines of the flow field are no longer well-behaved rather chaotic**. This type of flow is known as **turbulent flow**.

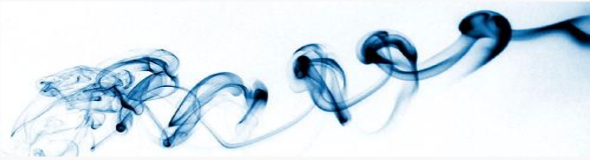


(*a*) Laminar flow



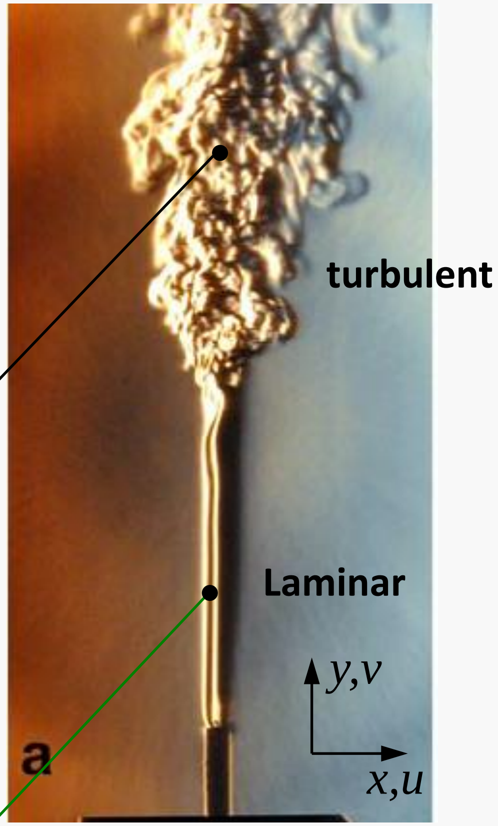
(*b*) Turbulent flow

Laminar and Turbulent Flows



The fundamental difference between laminar and **turbulent flow** lies in the **chaotic, random behavior of the various flow properties**. Such variations might occur in the three components of velocity (u, v, w), the pressure, the shear stress, the temperature, and any other variable that has a field description (such as density).

A typical time trace of the axial component of velocity (v) measured at a location in the flow is shown in the figure. **Its irregular, random nature is the distinguishing feature of turbulent flow.**



Candle plume

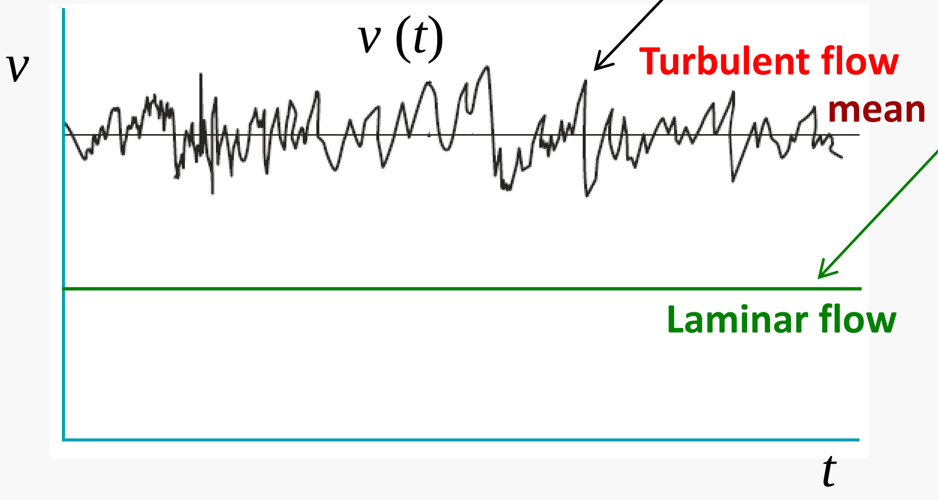
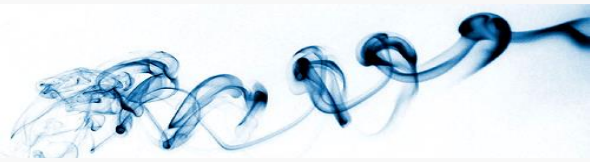


Fig. time trace of v-velocity component, $v = v(t)$ in flow

Laminar and Turbulent Flows

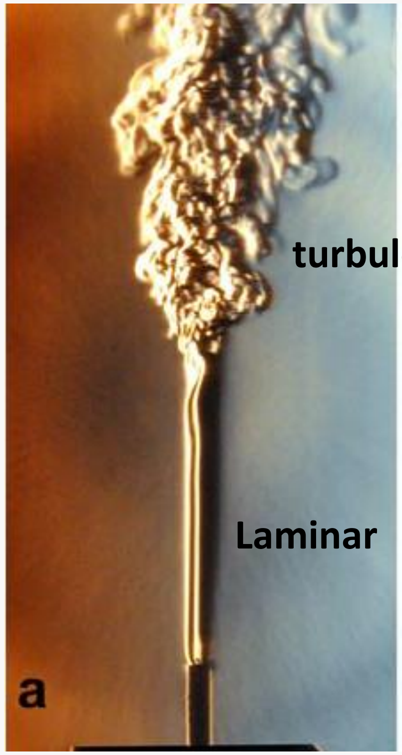


Shear stress in fluid flow (for **1-D flow**):

$$\text{Laminar}:\quad \tau_{\text{lam}} = \mu \frac{\partial u}{\partial y} \qquad ; \frac{\partial u}{\partial y} = \text{velocity gradient (1-D)}$$

$$\text{Turbulent}:\quad \tau_{\text{tur}} = (\mu + \mu_t) \frac{\partial u}{\partial y}$$

where μ is the **molecular (absolute) viscosity of fluid** and μ_t is the turbulent (eddy) **viscosity of flow**.



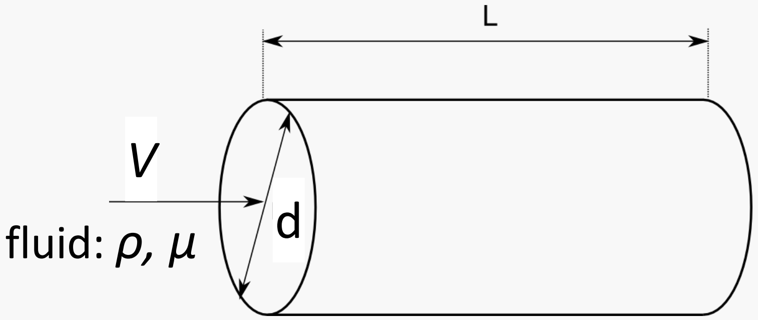
Candle plume

The nature of the flow (laminar/turbulent) can be characterized based of one dimensionless number called the "**Reynolds number, Re**".

As general criteria:

For flow through smooth pipe in ideal uniform conditions:

$\text{Re}_d < 2300$; flow is laminar
$\text{Re}_d > 4000$; flow is turbulent



; Re_d = Reynolds number based on pipe diameter $\text{Re}_d = \dfrac{\rho V d}{\mu}$

Laminar and Turbulent Flows

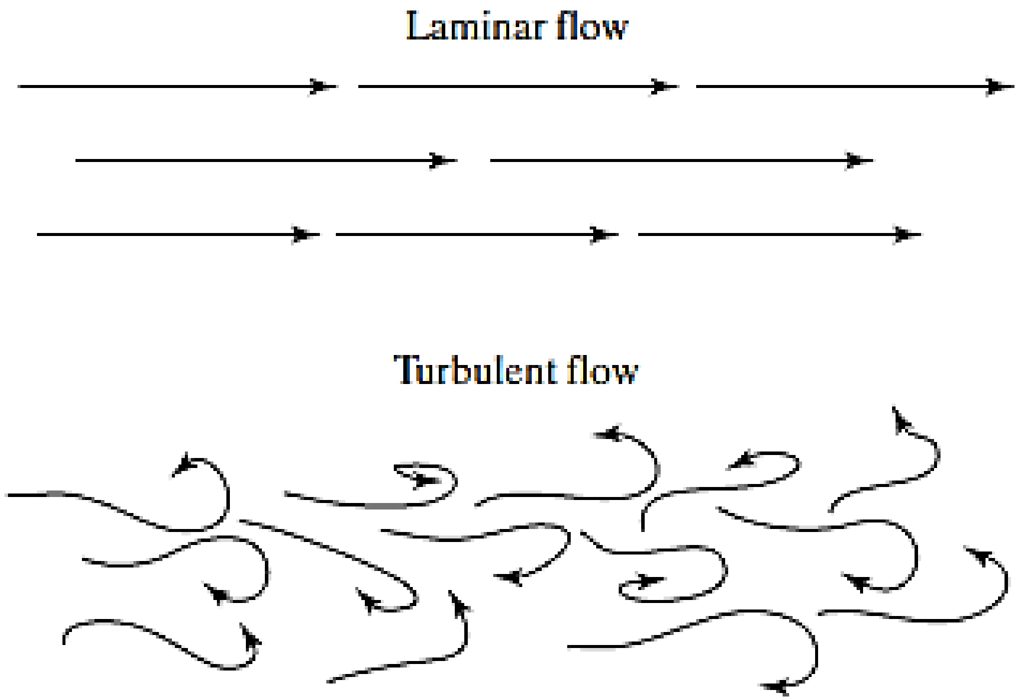


Figure 1.1. Schematic description of laminar and turbulent flows having the same average velocity.

Attached & separated flows

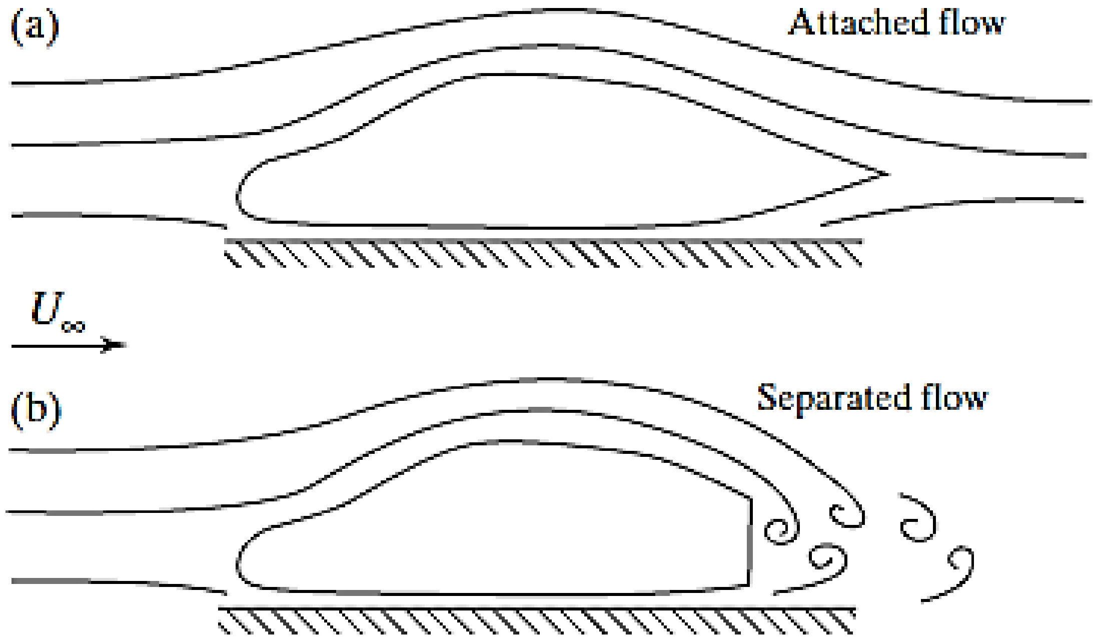
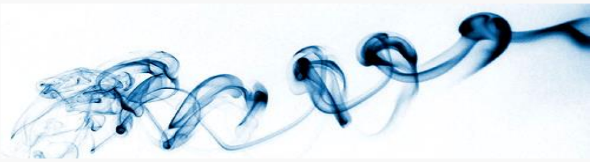


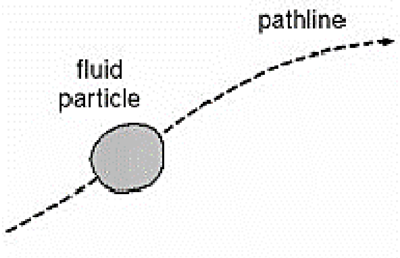
Figure 1.2. (a) Attached flow over a streamlined car and (b) the locally separated flow behind a more realistic automobile shape.

Eulerian and Lagrangian flow description



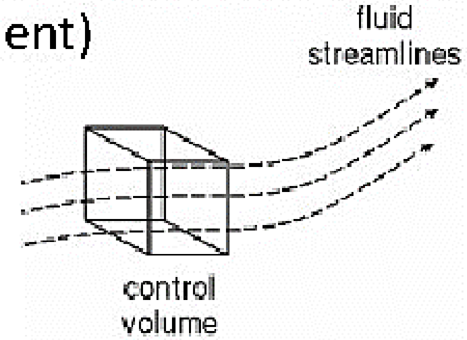
Lagrangian (follow the particle)

- A flow field can be thought of as being comprised of many "fluid particles". Mathematical laws can be derived for each fluid particle



Eulerian (fixed location)

- A flow field can be thought of in terms of how flow properties change at a fixed point in space and time (i.e. in a control volume or fluid element)

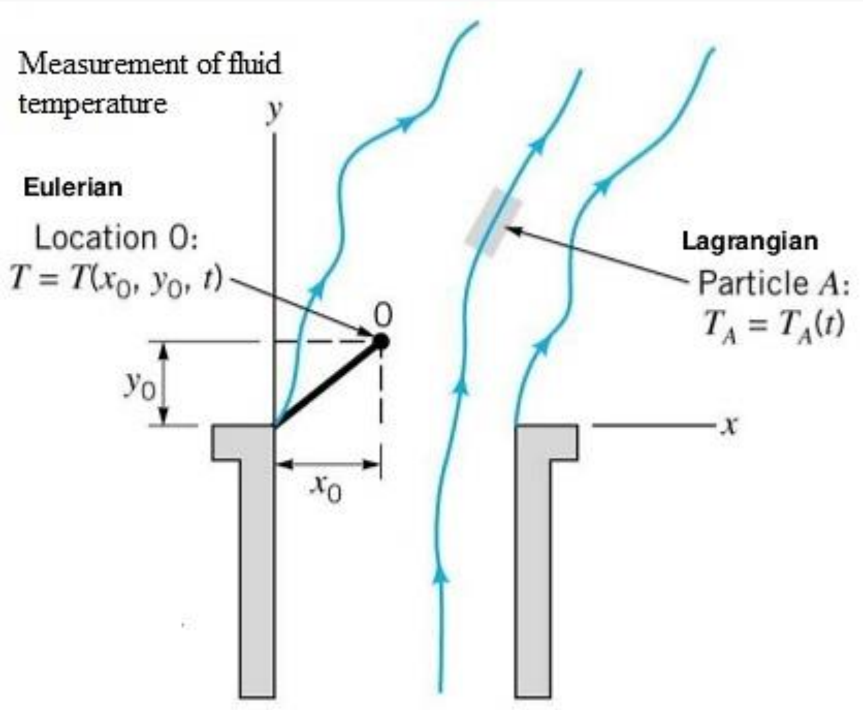


Particle description

Field description

(velocity field, pressure field, etc.)

Eulerian approach is more convenient for analysis in elementary fluid dynamics.



One-, Two-, and Three-Dimensional Flows

Generally, a fluid flow is a rather complex three-dimensional, time dependent phenomena-

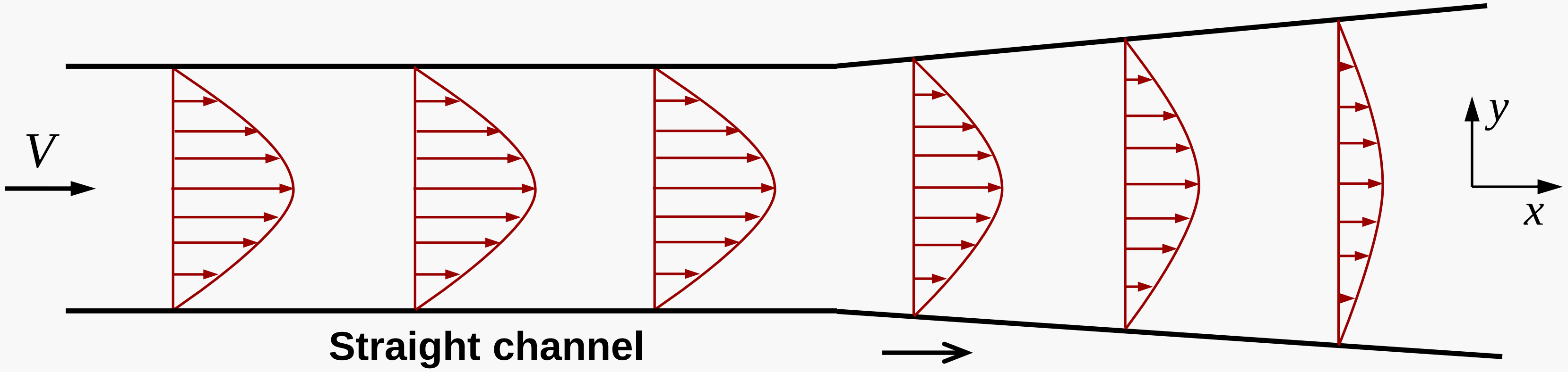
$$\vec{\mathbf{V}} = V(x, y, z, t)$$

In many situations, however, it is possible to make simplifying assumptions that allow a much easier understanding/analysis of the problem.

1-D flow: $\boldsymbol{V} = V(x)$

2-D flow: $\boldsymbol{V} = V(x, y)$

3-D flow: $\boldsymbol{V} = V(x, y, z)$

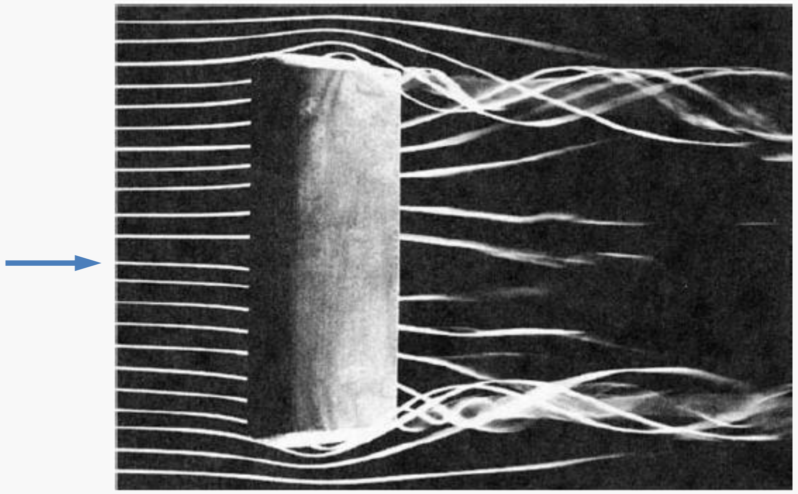


Straight channel

$$V = V(y)$$
$$V \neq V(x)$$

Diverging channel

$$V = V(x, y)$$



Complex 3-D flow over a model wing

Kinematics of Fluid Flow

Fluid kinematics is the analysis of fluid motion in space (x, y, z) and time (t) without being concerned with the actual forces necessary to produce the motion. It covers the analysis of velocity and acceleration of the fluid particle, and the description and visualization of its motion.

According to the "*continuum hypothesis*", the local velocity of fluid is the velocity of an infinitesimally small fluid particle (*composed of a large number of fluid molecules*) at a given instant. **It is generally a continuous function in space (x, y, z) and time (t)**.

Kinematics of Fluid Flow

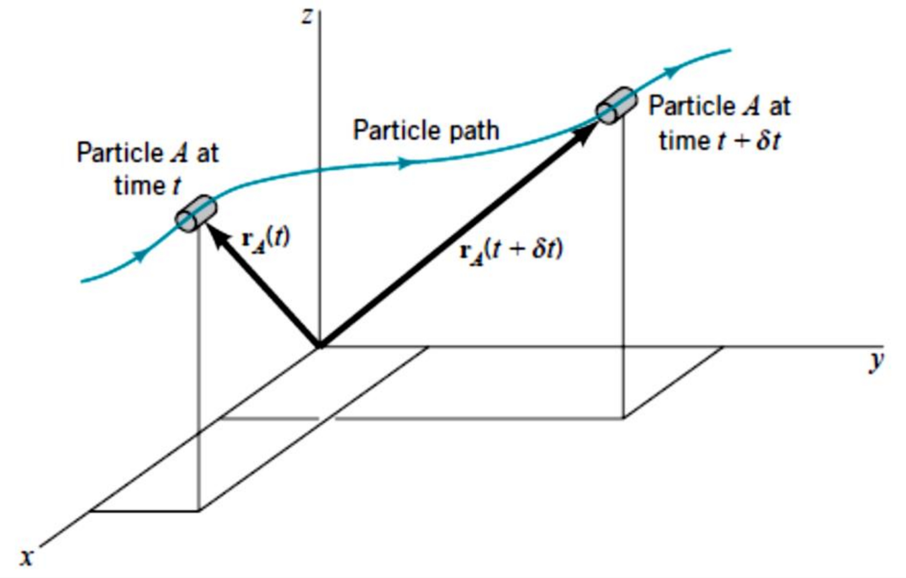
The infinitesimal small particles of a fluid are tightly packed together (continuum assumption). Thus, at a given instant in time, a description of any fluid property (such as density, pressure, velocity, acceleration etc.) may be given as a function of the fluid's location. This representation of fluid parameters as functions of the spatial coordinates (x, y, z, for example) is termed as ***field representation*** of the flow. The most important fluid variable is the ***velocity field***

$$\boxed{\vec{\mathbf{V}} = u(x, y, z)\,\hat{i} + v(x, y, z)\,\hat{j} + w(x, y, z)\,\hat{k}}$$

where u, v, w are the x, y, and z components of the velocity vector.

By definition, the velocity of a particle is the time rate of change of the position vector for that particle. As the figure shown:

$$\boxed{\frac{dr_A}{dt} = \mathbf{V}_A}$$



Kinematics of Fluid Flow

Of course, the specific field representation may be different at different times (unsteady flow).

By writing the velocity for all of the particles, the field description of the velocity vector can be obtained as,

$$\boxed{\vec{\mathbf{V}} = \mathbf{V}(x, y, z, t)}$$

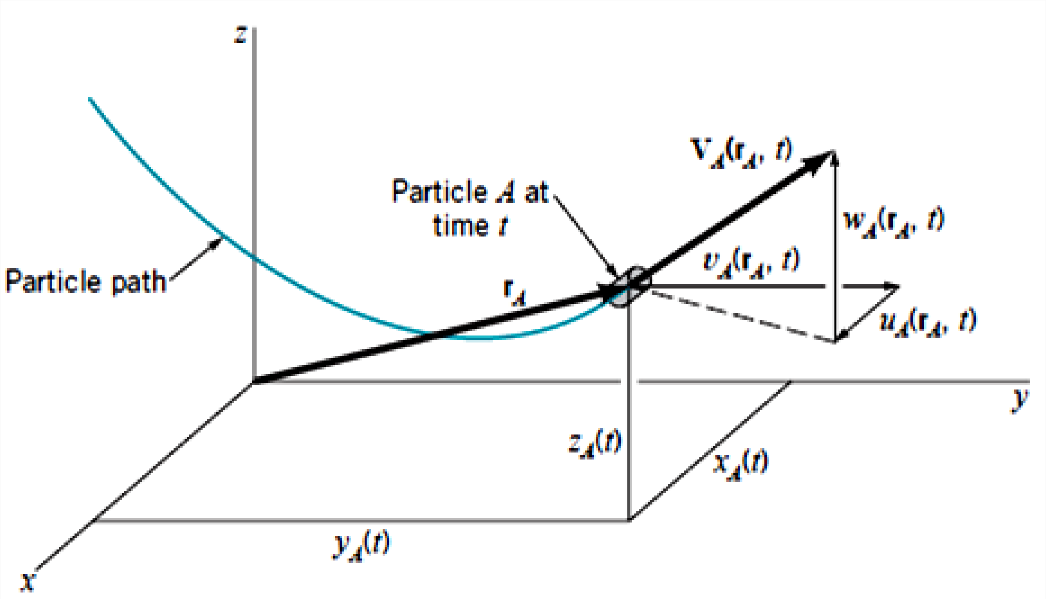
And the magnitude of velocity vector is,

$$\boxed{V = |\mathbf{V}| = \sqrt{u^2 + v^2 + w^2}}$$

Particle Acceleration

Consider a fluid particle moving along its pathline as shown in Fig. below. Particle A has velocity $\boldsymbol{V}_A$ which is a function of its location and the time as:

$$\boxed{\mathbf{V}_A = \mathbf{V}_A(r_A, t) = \mathbf{V}_A(x_A(t), y_A(t), z_A(t), t)}$$



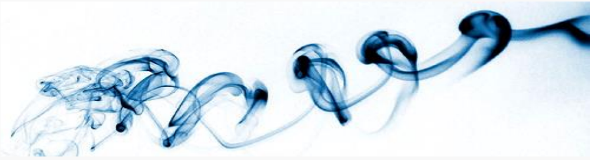
where $x_A = x_A(t)$, $y_A = y_A(t)$, and $z_A = z_A(t)$ define the locations of the moving particle.

Since the velocity may be a function of both position and time, its value may change because of the change in time as well as a change in the particle's position. Thus, the **acceleration of the particle** is written by **chain rule of differentiation**:

$$\mathbf{a}_A(t) = \frac{d\mathbf{V}_A}{dt}$$

$$= \frac{\partial \mathbf{V}_A}{\partial t} + \frac{\partial \mathbf{V}_A}{\partial x}\frac{dx_A}{dt} + \frac{\partial \mathbf{V}_A}{\partial y}\frac{dy_A}{dt} + \frac{\partial \mathbf{V}_A}{\partial z}\frac{dz_A}{dt}$$

Particle Acceleration



Using the fact that the particle velocity components are given by:

$$u_A = \frac{dx_A}{dt}, \quad v_A = \frac{dy_A}{dt}, \quad \text{and} \quad w_A = \frac{dz_A}{dt}$$

Then the acceleration of the fluid particle A comes as:

$$\mathbf{a}_A(t) = \frac{\partial \mathbf{V}_A}{\partial t} + \frac{\partial \mathbf{V}_A}{\partial x}\frac{dx_A}{dt} + \frac{\partial \mathbf{V}_A}{\partial y}\frac{dy_A}{dt} + \frac{\partial \mathbf{V}_A}{\partial z}\frac{dz_A}{dt}$$

$$\Rightarrow \mathbf{a}_A(t) = \frac{\partial \mathbf{V}_A}{\partial t} + u_A\frac{\partial \mathbf{V}_A}{\partial x} + v_A\frac{\partial \mathbf{V}_A}{\partial y} + w_A\frac{\partial \mathbf{V}_A}{\partial z}$$

Since the above is valid for any particle, the **acceleration field** can be written in general

$$\boxed{\mathbf{a} = \underbrace{\frac{\partial \mathbf{V}}{\partial t}}_{\text{Local acceleration}} + \underbrace{u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z}}_{\text{Convective acceleration}}}$$

Acceleration in Vector form

where $\mathbf{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$\mathbf{V} = u\hat{i} + v\hat{j} + w\hat{k}$

$$\left.\begin{aligned}
&\boxed{a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}} \\
&\boxed{a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}} \\
&\boxed{a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}}
\end{aligned}\right\} \text{Components of acceleration}$$

Particle Acceleration

The above result is often written in shorthand notation as

$$\mathbf{a} = \frac{D\mathbf{V}}{Dt}$$

where the operator

$$\boxed{\frac{D(\,)}{Dt} \equiv \frac{\partial(\,)}{\partial t} + u\frac{\partial(\,)}{\partial x} + v\frac{\partial(\,)}{\partial y} + w\frac{\partial(\,)}{\partial z}}$$

is termed as ***material derivative*** (***total, particle*** or ***substantial derivative***).

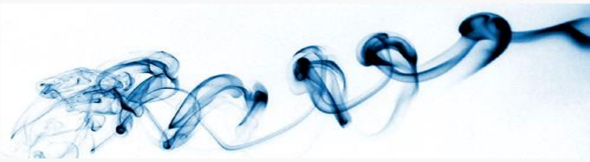
An often-used shorthand notation for the **material derivative** is

$$\boxed{\frac{D(\,)}{Dt} \equiv \frac{\partial(\,)}{\partial t} + (\mathbf{V}\cdot\nabla)(\,)}$$

Total Local Convective

where $\nabla = \text{gradient operator} = \frac{\partial}{\partial x}\,\hat{i} + \frac{\partial}{\partial y}\,\hat{j} + \frac{\partial}{\partial z}\,\hat{k}$

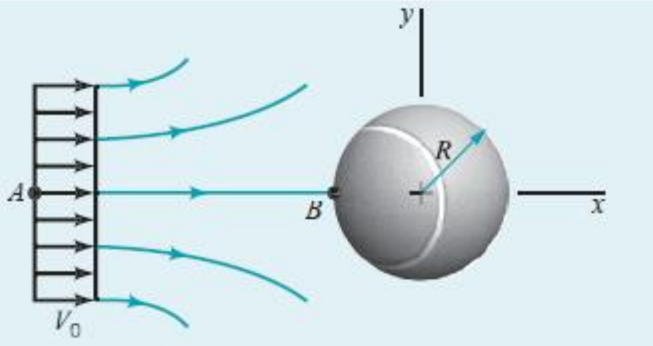
$$\mathbf{V}\cdot\nabla = \left(u\,\hat{i} + v\,\hat{j} + w\,\hat{k}\right)\cdot\left(\frac{\partial}{\partial x}\,\hat{i} + \frac{\partial}{\partial y}\,\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$



Problem

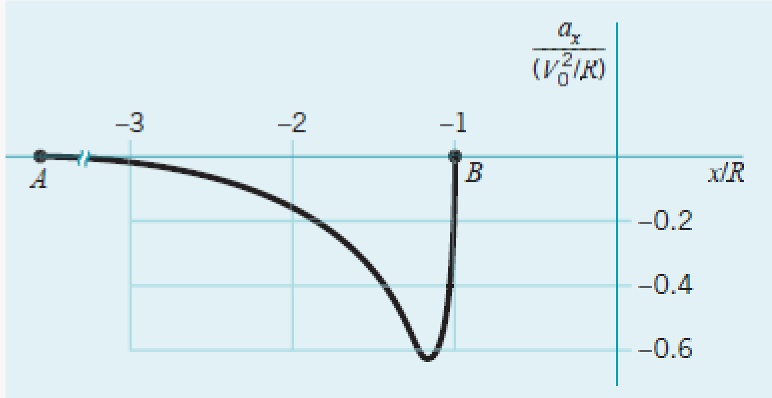
An incompressible, inviscid fluid flows steadily past a ball of radius R. The fluid velocity along the streamline A-B is given by

$$\mathbf{V} = V_0\left(1+\frac{R^3}{x^3}\right)\hat{i}$$



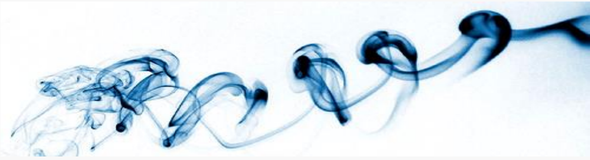
where V_0 is the upstream velocity far ahead of the sphere.

Determine the acceleration experienced by fluid particles as they flow along this streamline.



Answer:

$$a_x = -3\left(\frac{V_0^2}{R}\right)\frac{1+\left(\frac{R}{x}\right)^3}{\left(\frac{x}{R}\right)^4}$$



Problem

A simple approximate velocity and pressure fields for incompressible flow through a converging duct can be expressed as

$$\vec{V} = (U_0 + bx)\,\hat{\imath} - by\hat{\jmath}$$

$$P = P_0 - \frac{\rho}{2}[2U_0bx + b^2(x^2 + y^2)]$$

where U_0 and P_0 are respectively, the horizontal speed and pressure at x = 0 and b is a constant.

(a) Calculate the material acceleration for fluid particles passing through this duct.

(b) Generate an expression for the rate of change of pressure following a fluid particle.